Conceptual Model-based Problem Solving for Students with Learning Disabilities/Difficulties

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Presented at ICMI study 23, Panel on Special Education, Macau, June, 2015

Students with Disabilities and Problem Solving Instruction

- Historically, mathematics instruction in special education primarily focuses on rote memorization of facts and computational skills (Baroody & Hume, 1991; Bottge, 1999; Parmar, Cawley, & Miller, 1994; Woodward & Montague, 2000).

- Traditional problem solving methods (e.g., key word) have not led to positive outcomes

  - Example:
  - Jill gave away 6 cookies in the morning. She <u>gave away</u> 2 cookies in the afternoon. How many cookies did she give away that day?" (Kelly & Carnine, 1996, p.5)

Contemporary Education Climate in the U.S.

- Problem solving is central to mathematics reform and articulated in the NCTM (2000) Principles and Standards for School Mathematics (De Corte, Greer, & Verschafel, 1996).
- The Individuals with Disabilities Act (IDEA, 1997, 2004) require that students with disabilities be provided not just access, but meaningful access to the general education curriculum.
- The Adequate Yearly Progress component of No Child Left Behind (NCLB, 2002) has mandated that certain percentages of students with disabilities achieve proficiency on state assessments.
- The Common Core Standards (2012)

Contemporary Education Climate

- Common Core Standards (CCS, Council of Chief State School Officers and National Governors Association, 2010) endorse a focused and coherent curriculum.
- These standards heavily emphasize conceptual understanding of ideas and the connections between mathematical ideas (Common Core State Standards Initiative [CCSSI], 2012).
- The CCS emphasize "model with mathematics."
- The CCS emphasize higher order thinking and reasoning as well as algebra readiness throughout elementary mathematics

Background and Theoretical Framework

- As the outcome of a collaborative work that integrates research-based practices from math education and special education, we have developed an intelligent tutor, PGBM-COMPS, that emphasizes conceptual understanding of multiplicative reasoning at both concrete and symbolic levels.

- The intelligent tutor draws on three research-based frameworks: data (or statistical) learning from computer sciences, a constructivist view of learning from mathematics education, and Conceptual Model-based Problem Solving (COMPS, Xin, 2012) that generalizes word-problem underlying structures (WP *story-grammar*) from special education.

Nurturing Multiplicative Reasoning In Students With Learning Disabilities In A Computerized Conceptual-Modeling Environment (NMRSD-CCME) [1]

Project Principal Investigators

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[1] This research is supported by the US National Science Foundation under grant DRL 0822296. The opinions expressed do not necessarily reflect the views of the Foundation.

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PGBM-COMPS intelligent tutor

PGBM

(Tzur et al., 2013):

"Please go and bring me…" Cubes and Towers Game

- Establish fundamental mathematical ideas that is critical to MR:
- Understand the concepts of
  - Composite Unit (CU) and
  - Multiplicative Double Counting (mDC)

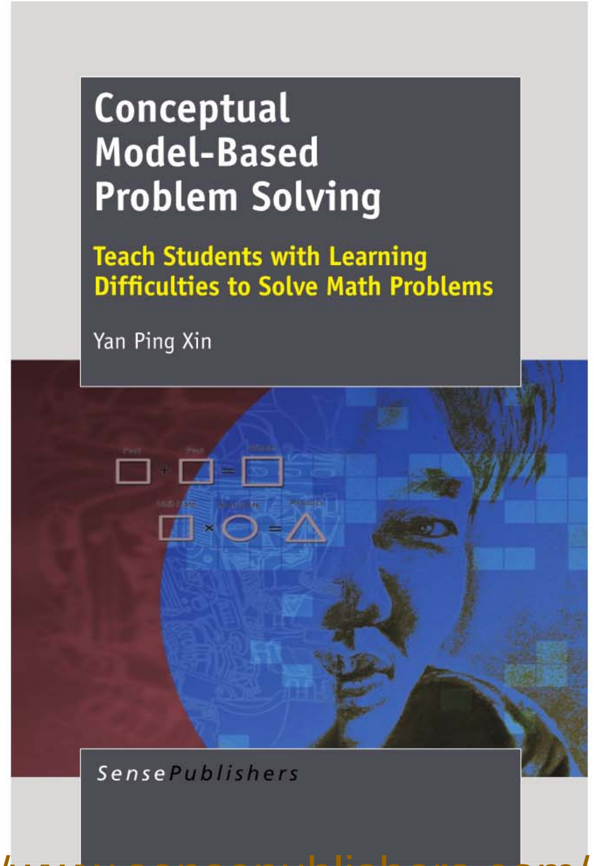
COMPS

(Xin, 2012):

Conceptual model-based problem solving

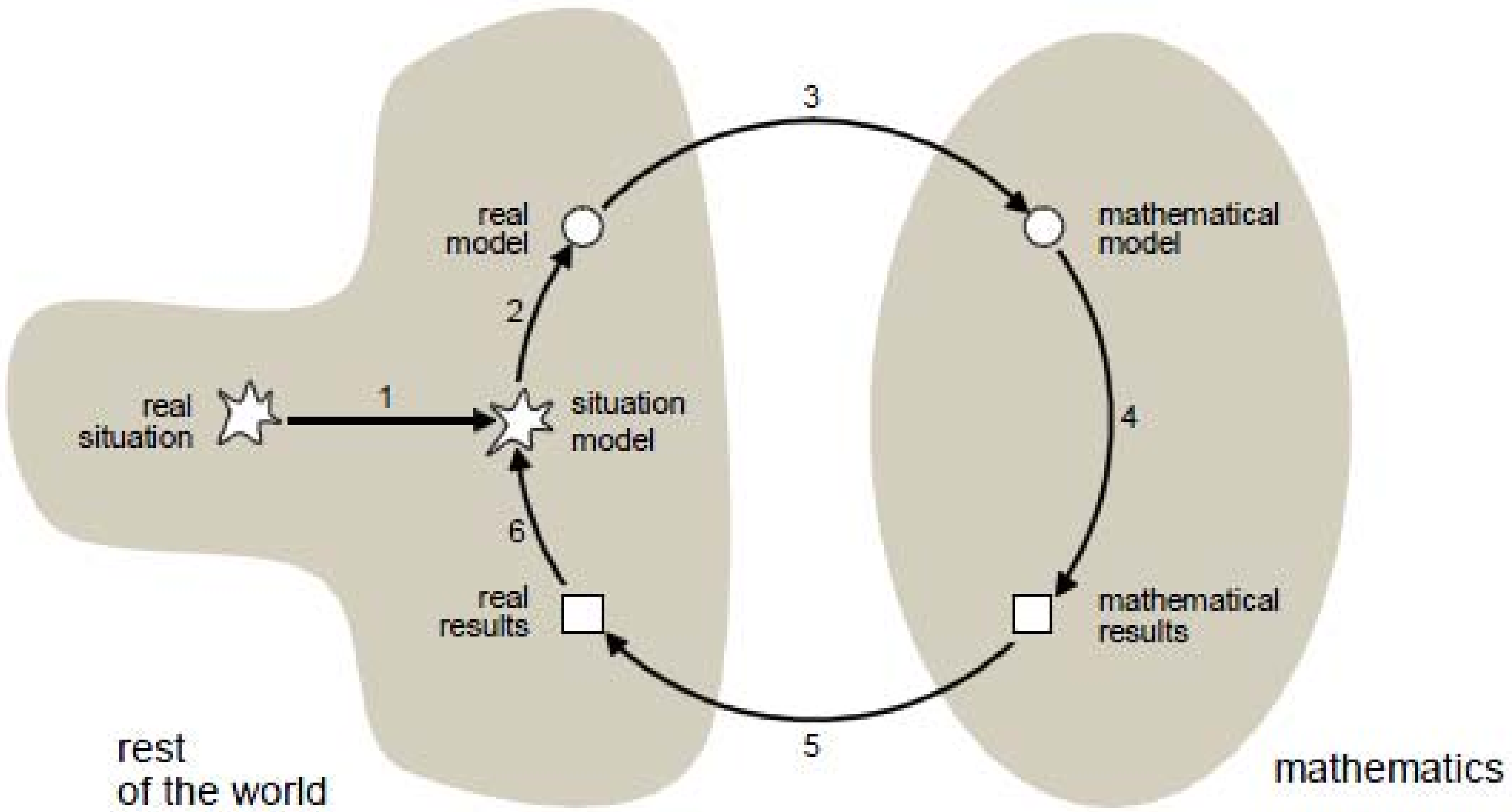
- Make the connection between the concrete model and the symbolic mathematical model/equation (make explicit the reasoning behind MR)
- Facilitate the "mental leap" from real world situated model to mathematical model for solution

Xin, Y. P. (2012). *Conceptual model-based problem solving: Teach students with learning difficulties to solve math problems*. Sense publishers, Boston.



Link to the book: https://www.sensepublishers.com/catalogs/bookseries/other-books/conceptual-model-based-problem-solving/

Modeling Cycle (Blum & Leiss, 2005)



Modeling Cycle

- Many students have difficulties in making the transition from a real situational model to a mathematical model;
- It is a weak area in students' mathematical understanding (Blomhøj, 2004).
- While the PGBM part of the program aimed to establish fundamental mathematical ideas through concrete modeling, the COMPS program attempted to facilitate the transition from real-world, situational model to mathematical model.

PGBM-COMPS Intelligent Tutor

PGBM	Module	COMPS
PGBM MDC: $6T_3$ How many cubes in all? SUC : $6T_3$ vs. $2T_3$ who has more towers?	**Module A**	**COMPS** Equal Group (EG) problem structure instruction EG: product unknown
PGBM **UDS:** $6T_3 + 6$ cubes: how many cubes in all?	**Module B**	**MUC:** $6T_3 + 12$ cubes = $?T_3$
PGBM QD: Given 36 cubes, how many towers of 9 can you make?	**Module C**	**COMPS** EG: #of units unknown and mixed problems
PGBM PD: Given 60 cubes; put them in 3T How many cubes in each tower?	**Module D**	**COMPS** EG: unit rate unknown and mixed problems
MC: Ray has 6 books, and Joe has 8 times as many books as Ray. How many books does Joe have?	**Module E**	**COMPS** Multiplicative Compare (MC) and mixed problem solving

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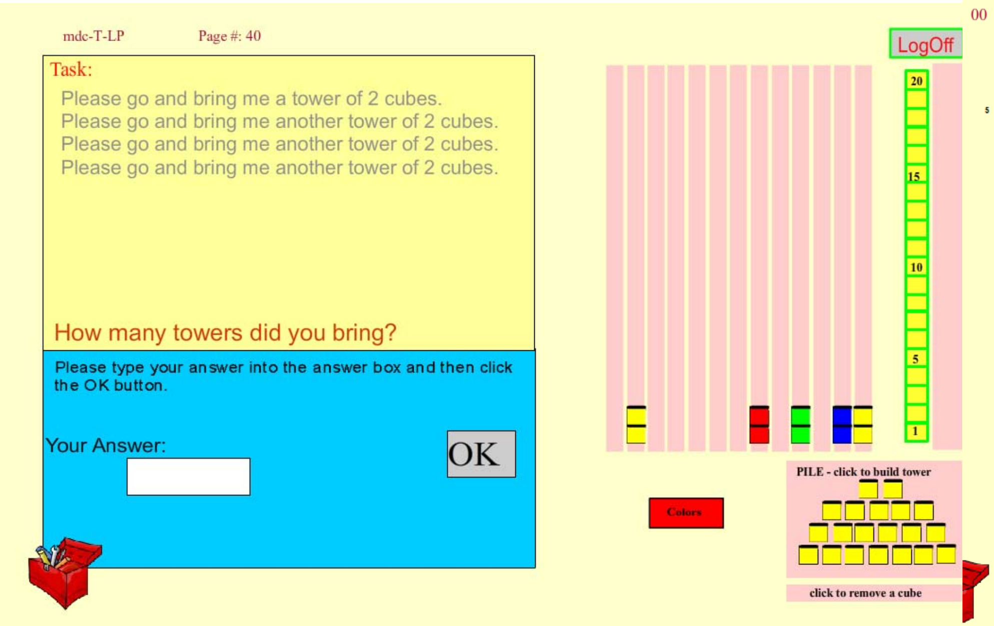
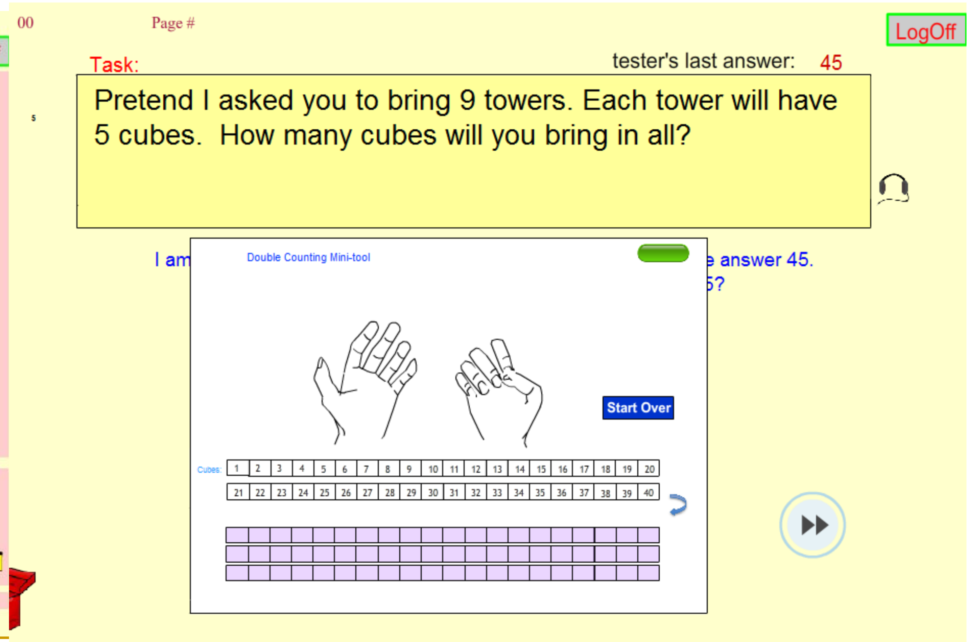
Module A "PGBM" Game

- **mDC** - Multiplicative Double Counting

PGBM 7T$_2$; how many cubes in all? 7 x 2

Pretend PGBM 9T$_5$; how many cubes in all? 9 x 5





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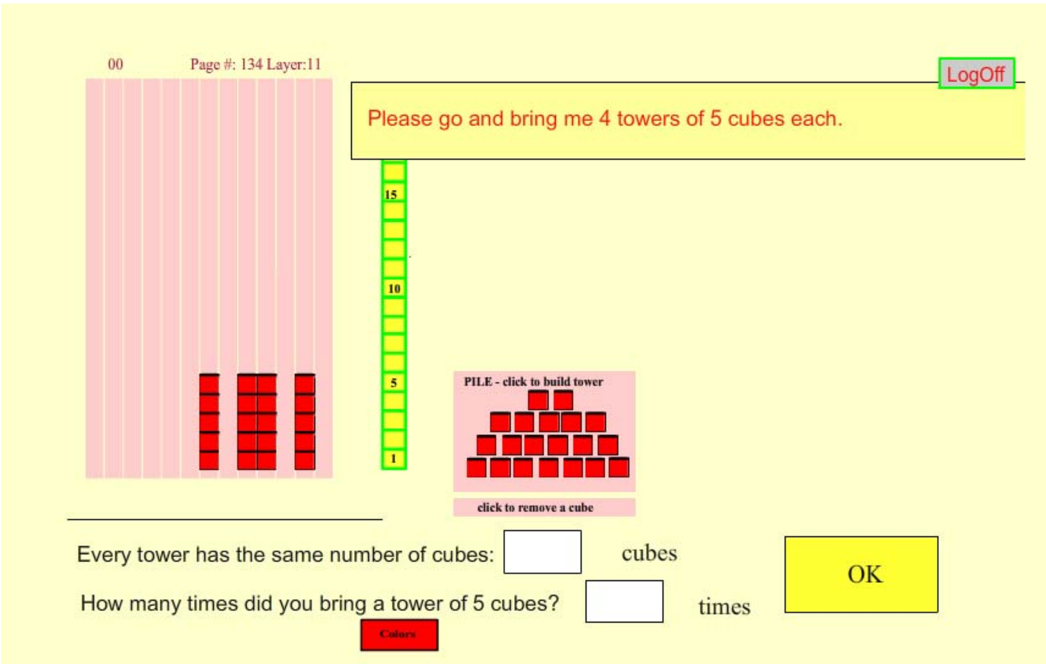
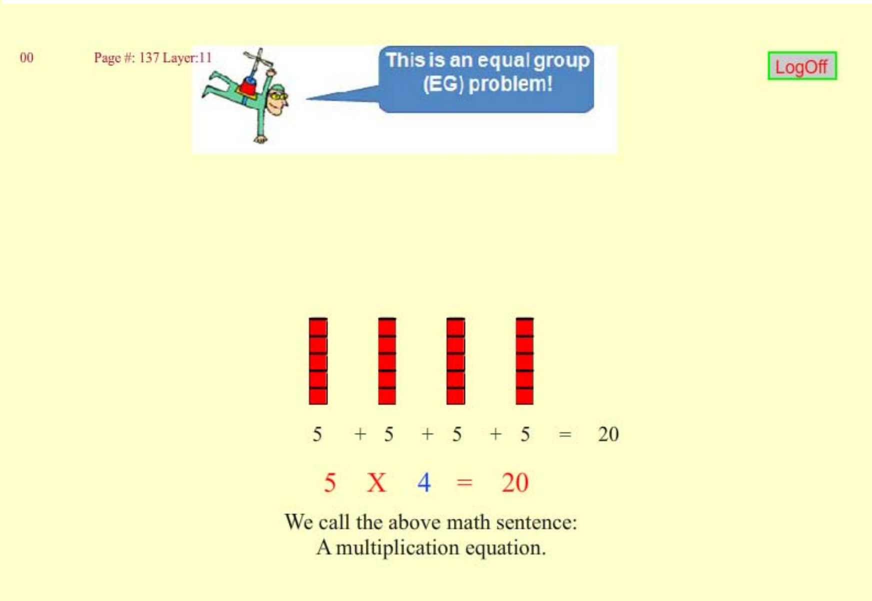
Module A **COMPS**

- **COMPS- mDC**
  - Use COMPS model to solve various situated product unknown problems involving large quantities

UR x # Units = ?





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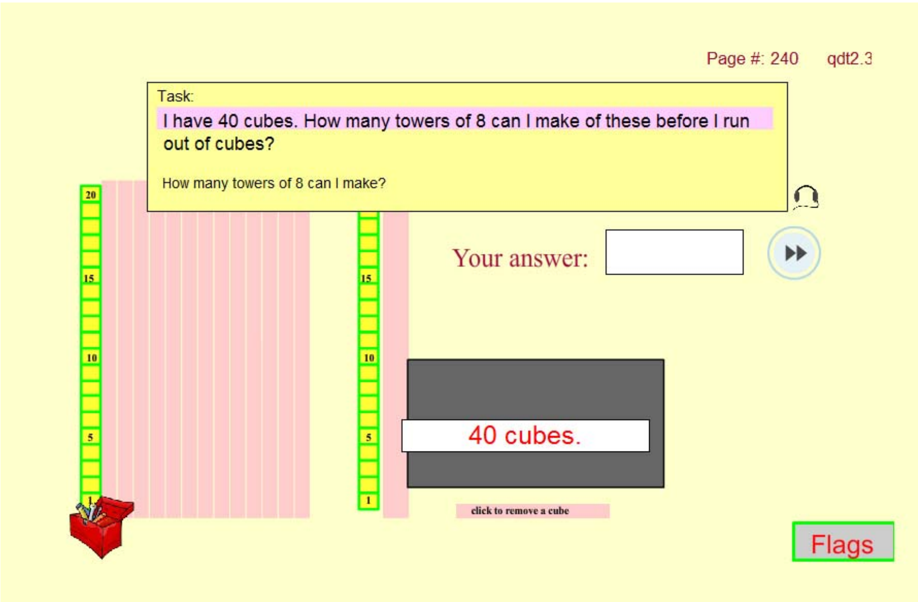
Module C PGBM

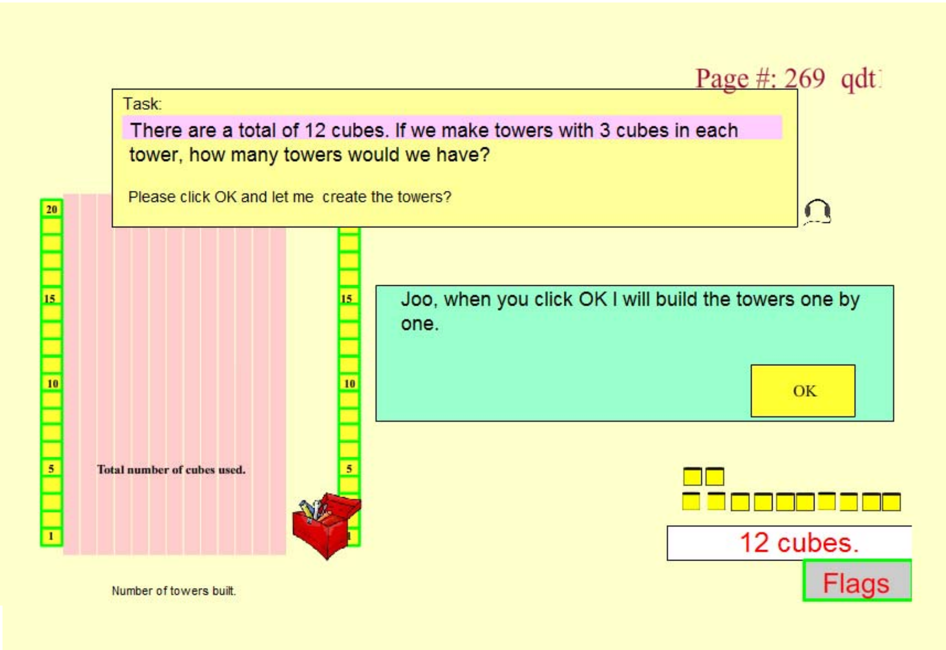
- **QD**

  -mDC-QD: Unit Segmenting

  -dCSC (division Concept-Symbol Coordination): 40/8

  Given 40 cubes, how many towers of 8 can I make?





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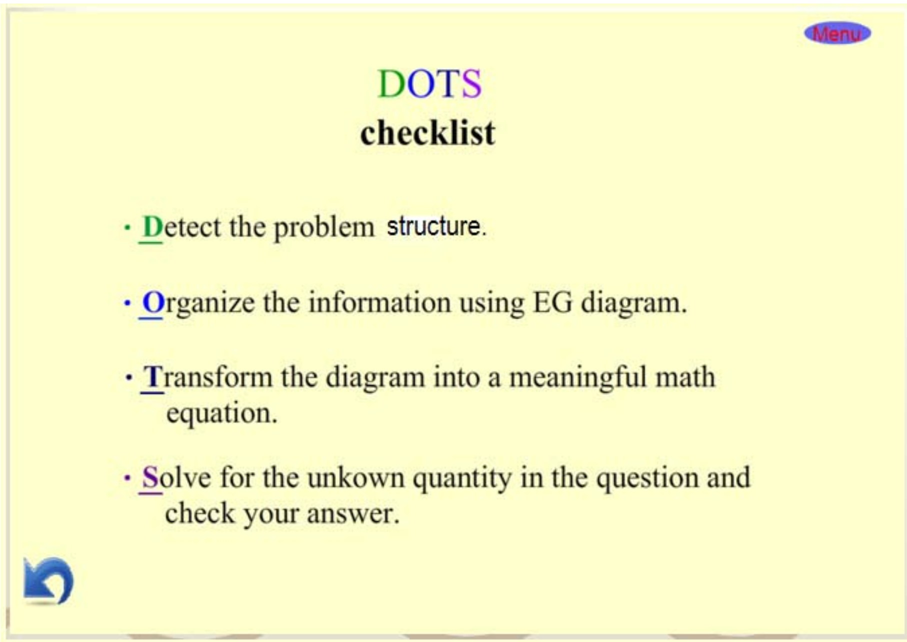
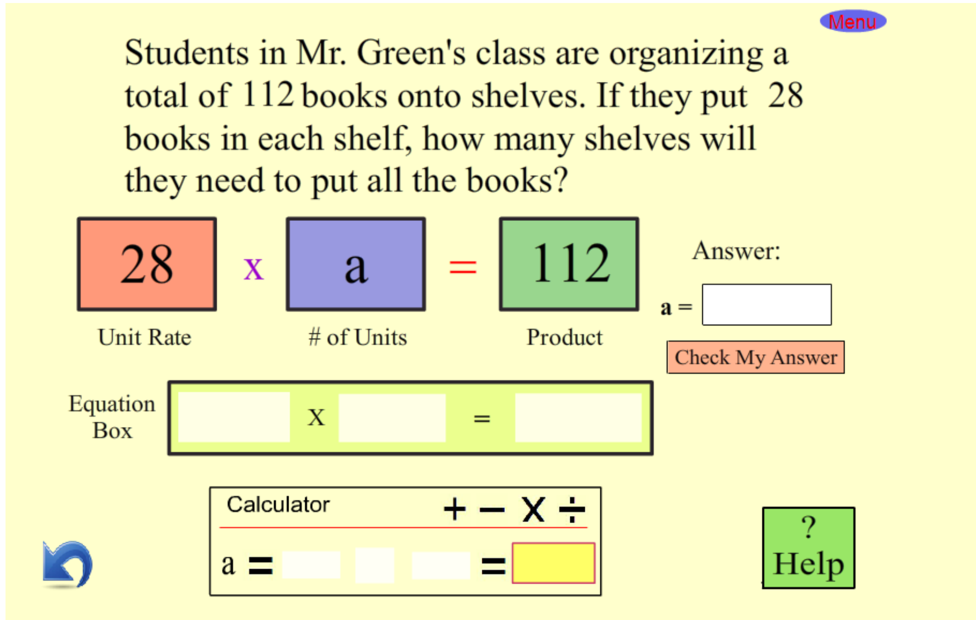
COMPS Model (Xin, 2012)

- Introduce DOTS Checklist
- Solve more complex real-world problems using conceptual model-based diagram equations.





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Xin, Y. P., Tzur, R., Si, L., Hord, C., Liu, J., Park, J. Y, Cordova, M., & Ruan, L. Y. (2013, April). A Comparison of Teacher-delivered Instruction and an Intelligent Tutor-assisted Math Problem-Solving Intervention Program. Paper presented at the 2013 AERA, San Francisco, CA.

- The purpose of this study was to compare the effectiveness of the computer-assisted PGBM-COMPS tutoring system (PGBM-COMPS) with school teacher-delivered instruction (TDI) on enhancing the multiplicative reasoning (MR) and problem solving skills of students with LDM.

- Results indicated that there is a statistically significant Group-Time *Interaction* effect (MR: $p < .01$; COMPS: $p < .01$): The improvement rate of the PGBM-COMPS group is much greater than that of the TDI group (on MR measure: Effect Size [ES] = 2.14; on COMPS measure: ES = 2.26)

Figure 1: Performance of the two groups (1= PGBM-COMPS, 2 = TDI) on the MR test before (Time = 1) and after the intervention (Time = 2, 3, & 4)

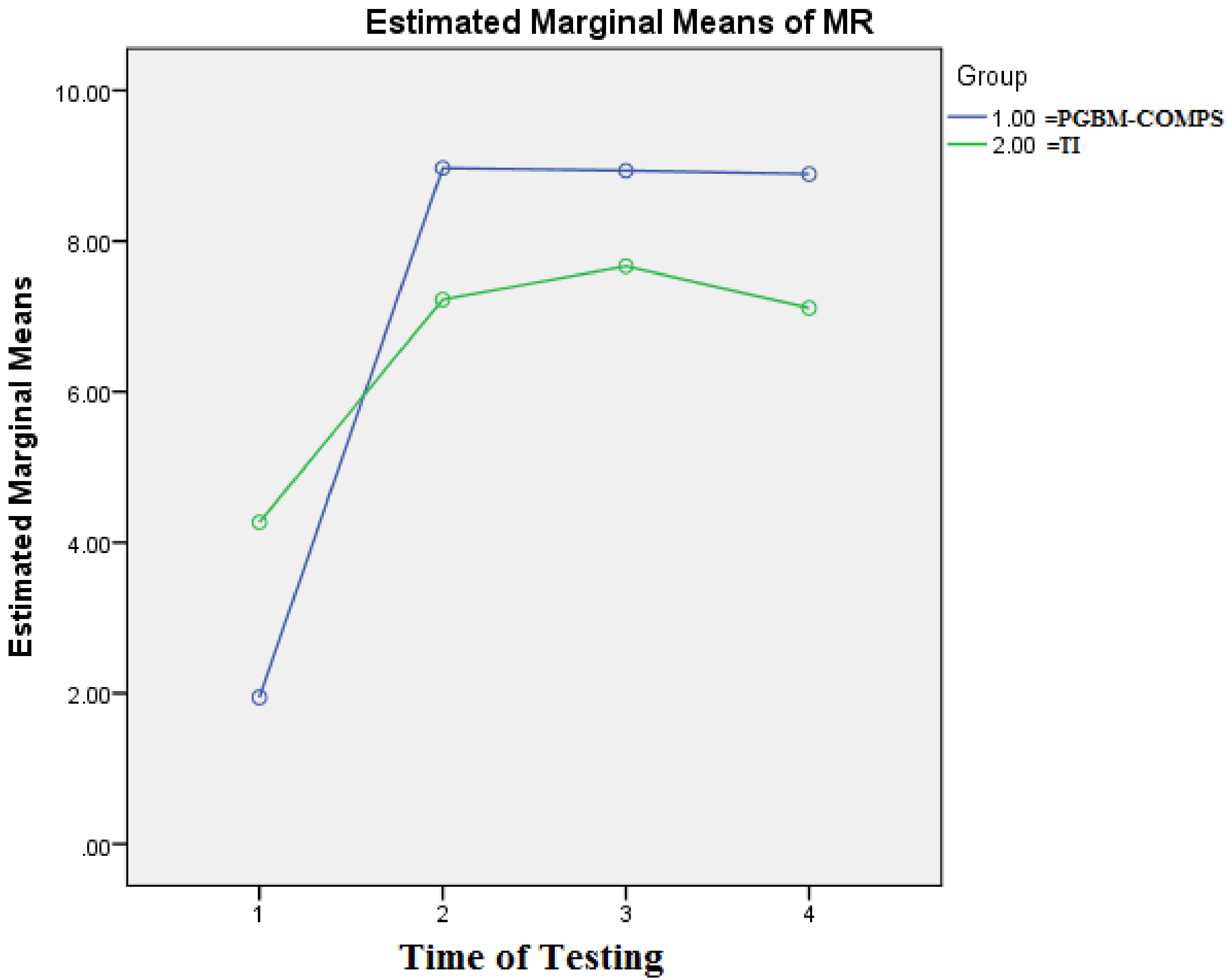
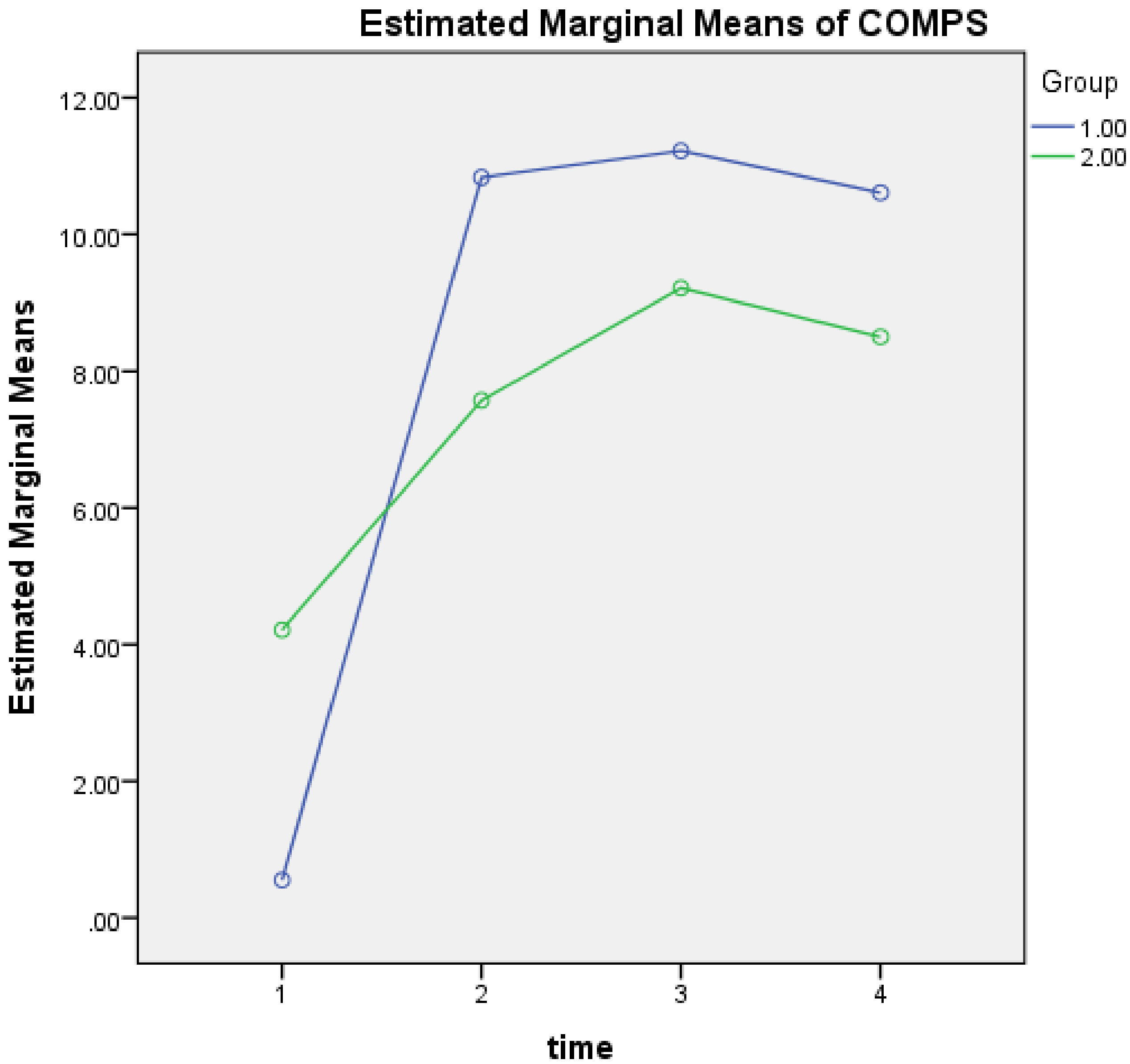


Figure 2: Performance of the two groups (1= PGBM-COMPS, 2 = TDI) on the COMPS test before (Time = 1) and after the intervention (Time = 2, 3, &4))



COMPS Test: Sample items

Table 2b

Sample Problems in Probes: Multiplication/ Division (from Xin et al., 2008, p. 168)

Problem type	Sample Problem Situations
Equal Group	
Unit Rate unknown	A school arranged a visit to the museum in Lafayette Town. It spent a total of $667 buying 23 tickets. How much does each ticket cost?
Number of units (sets) unknown	There are a total of 575 students in Centennial Elementary School. If one classroom can hold 25 students, how many classrooms does the school need?
Product unknown	Emily has a stamp collection book with a total of 27 pages, and each page can hold 13 stamps. If Emily filled up this collection book, how many stamps would she have?
Multiplicative Compare	
Compared set unknown	Cameron has 242 marbles. Cameron has a brother named Isaac. Cameron has 22 times as many marbles as Isaac. How many marbles does Isaac have?
Referent set unknown	Gina has sent out 462 packages in the last week for the post office. Gina has sent out 21 times as many packages as her friend Dane. How many packages has Dane sent out?
Multiplier unknown	It rained 147 inches in New York one year. In Washington D.C., it only rained 21 inches during the same year. The amount of rain in New York is how many times the amount of rain in Washington D.C.?

Results on Far Transfer Measure

- As for the SAT (Stanford Achievement Test) far-transfer measure, using pretest to posttest *gain* score as the measure, Independent Samples T-test analysis indicates a significant difference between the two groups (COMPS: M_{gain} = 10.22, SD=8.27; TDI: M_{gain}= 1.86, SD=1.39; p = .02) favoring the COMPS group (ES = 1.41).

Significance of the Study

- Give that the Common Core demands much deeper content knowledge from teachers of mathematics (CCSSI, 2012), the preliminary findings of this study is encouraging.

- The PGBM-COMPS intelligent tutor, which integrates the best practices from general math education and special education, seems to yield better outcomes in multiplicative problem solving.

- Through the integration of heuristic instruction (that facilitates concept construction) and the explicit model-based problem-solving instruction, it seems that the PGBM-COMPS programs have promoted generalized problem-solving skills of students with LDM.

- This was reflected in students' improved performance on the MR and COMPS criterion tests, and more importantly, on the far transfer measure, SAT, a norm-referenced standardized test.

Concluding Remarks

- Design the teaching/instruction that is built on individual student's learning profile
- Collect common student responses (e.g., types of thinking) from experimental data
- Attempt to integrate reasoning and problem-solving competences
- Invite 'constructivism' into special education discourse – talk about the 'black-box' known as LEARNING
- Challenge: how to make assessment and teaching more intelligent

Any Questions? Pls contact

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